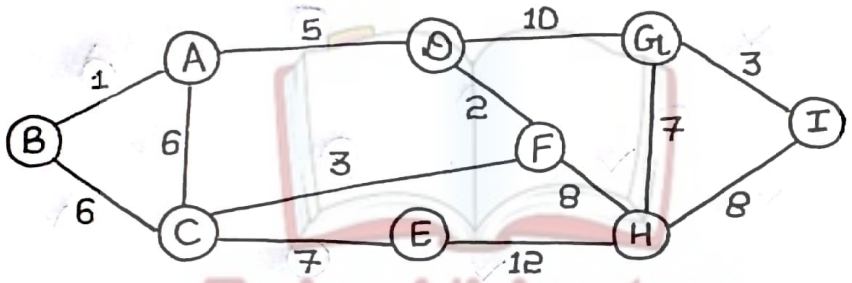
Gate Problems...

Question:

For the undirected, weighted graph given below, which of the following sequences of edges represents a correct execution of Prim's Algorithm to construct a minimum spanning tree?

(GATE-2008)



(A) (a,b), (d,f), (f,c), (g,i), (d,a), (g,h), (c,e), (f,h)

(B) (c,e), (c,f), (f,d), (d,a), (a,b), (g,h), (h,f), (g,i)

(C) (d,f), (f,c), (d,a), (a,b), (c,e), (f,h), (g,h), (g,i)

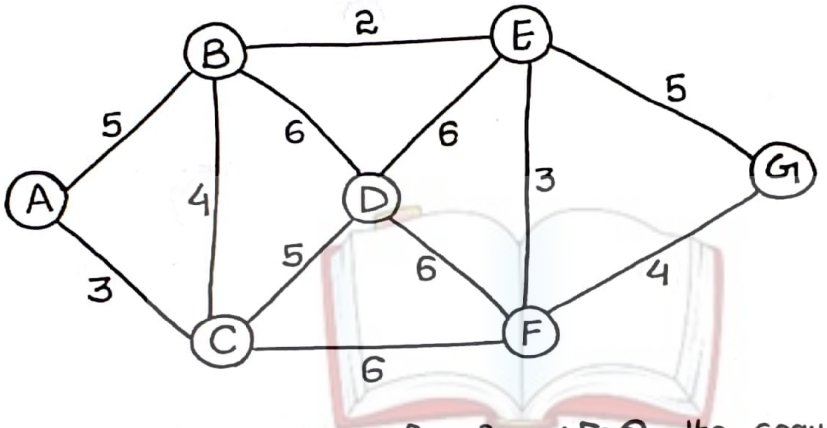
(D) (h,g), (g,i), (h,f), (f,c), (f,d), (d,a), (a,b), (c,e)

Solution:

Correct Option is (C)

Question:

Consider the following graph- (GATE-2009)



Which one of the following is NOT the sequence of edges added to the minimum spanning tree using Kruskal's Algorithm?

(A) (b,e), (e,f), (a,c), (b,c), (f,g), (c,d)

(B) (b,e), (e,f), (a,c), (f,g), (b,c), (c,d)

(C) (b,e), (a,c), (e,f), (b,c), (f,g), (c,d)

(D) (b,e), (e,f), (b,c), (a,c), (f,g), (c,d)

Solution:

Correct option is (D)

Question:

Consider a complete undirected graph with vertex set $\{0, 1, 2, 3, 4\}$.

Entry W_{ij} in the matrix W below is the weight of the edge (i, j)

(GATE-2010)

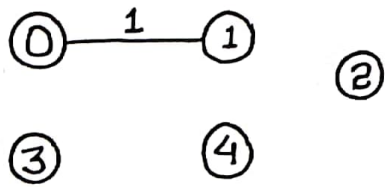
$$W = \begin{bmatrix} 0 & 1 & 8 & 1 & 4 \\ 1 & 0 & 12 & 4 & 9 \\ 8 & 12 & 0 & 7 & 3 \\ 1 & 4 & 7 & 0 & 2 \\ 4 & 9 & 3 & 2 & 0 \end{bmatrix}$$

Find the weight of the minimum spanning Tree (MST).

Solution:

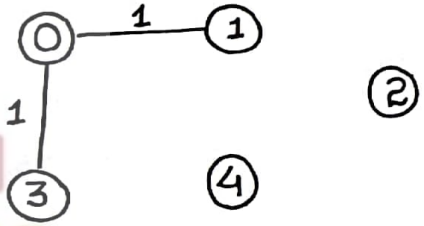
Step-1 :

	0	1	2	3	4
0	0	(1)	8	1	4
1	(1)	0	12	4	9
2	8	12	0	7	3
3	1	4	7	0	2
4	4	9	3	2	0



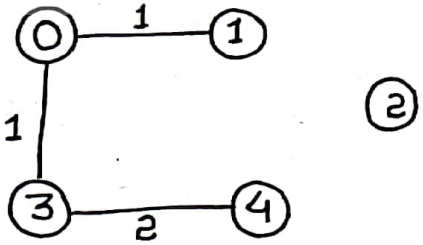
Step-2 :

	0	1	2	3	4
0	0	(1)	8	(1)	4
1	(1)	0	12	4	9
2	8	12	0	7	3
3	(1)	4	7	0	2
4	4	9	3	2	0



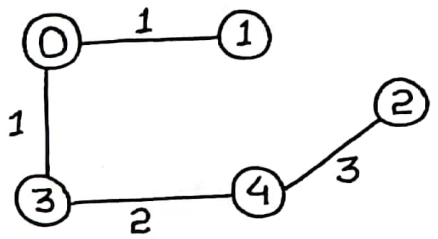
Step-3 :

	0	1	2	3	4
0	0	(1)	8	(1)	4
1	(1)	0	12	4	9
2	8	12	0	7	3
3	(1)	4	7	0	(2)
4	4	9	3	(2)	0



Step-4:

	0	1	2	3	4
0	0	(1)	8	(1)	4
1	(1)	0	12	4	9
2	8	12	0	7	(3)
3	(1)	4	7	0	(2)
4	4	9	(3)	(2)	0



Minimum Spanning Tree (MST)

Weight of MST = 1 + 1 + 2 + 3

= 7 units

Question:

Consider a complete undirected graph with vertex set $\{0,1,2,3,4\}$. Entry W_{ij} in the matrix W below is the weight of the edge $\{i,j\}$

$$W = \begin{bmatrix} 0 & 1 & 8 & 1 & 4 \\ 1 & 0 & 12 & 4 & 9 \\ 8 & 12 & 0 & 7 & 3 \\ 1 & 4 & 7 & 0 & 2 \\ 4 & 9 & 3 & 2 & 0 \end{bmatrix}$$

(GATE-2010)

What is the minimum possible weight of a spanning tree T in this graph such that vertex 0 is a leaf node in the tree T?

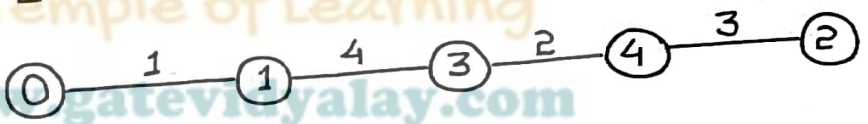
Solution:

Solution:

To find the required spanning tree, we will select only one minimum weight from row-1 of vertex-0 thus making sure that it does not get connected to any other vertex because leaf node must be connected to only one vertex of the tree. Rest, the method would be same.

So, we have-

	0	1	2	3	4
0	0	(1)	8	1	4
1	(1)	0	12	(4)	9
2	8	12	0	7	(3)
3	1	(4)	7	0	(2)
4	4	9	(3)	(2)	0



∴ Minimum possible weight of spanning tree T

= 1 + 4 + 2 + 3

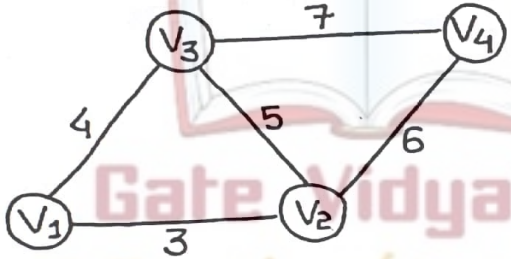
= 10 units

Note: It is not MST.

Question:

An undirected graph $G(V,E)$ contains n ($n>2$) nodes named $v_1, v_2, \ldots, v_n$. Two nodes v_i, v_j are connected if and only if $0 < |i-j| \leq 2$. Each edge (v_i, v_j) is assigned a weight $i+j$. A sample graph with $n=4$ is shown below-

(GATE-2011)



What will be the cost of the Minimum Spanning Tree (MST) of such a graph with n nodes ?

(A) $\frac{1}{12}(11n^2-5n)$

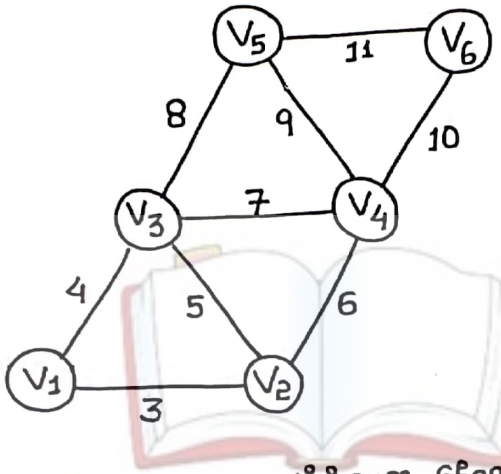
(B) n^2-n+1

(C) $6n-11$

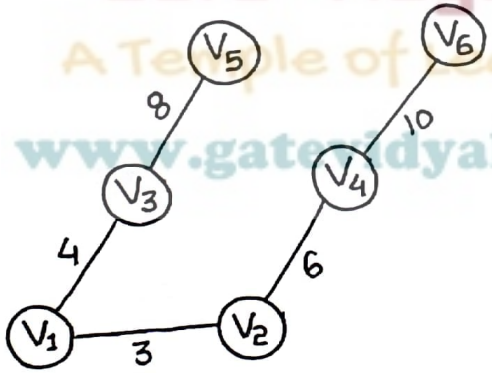
(D) $2n+1$

Solution :

Consider $n = 6$, then the graph can be drawn as -



Now Let us draw the Minimum Spanning Tree -



∴ Weight of MST = 31

Only option (B) gives value 31 on substituting $n = 6$

∴ Option (B) is correct.

Question:

Consider a weighted complete graph G on the vertex set $\{v_1, v_2, \ldots, v_n\}$ such that the weight of the edge (v_i, v_j) is $2|i-j|$. The weight of a minimum spanning tree of G is ____?

(GATE-2006)

(A) $n-1$

(B) $2n-2$

(C) ${}^nC_2$

(D) 2

Solution:

Try yourself!

Consider some value of n and check the options.

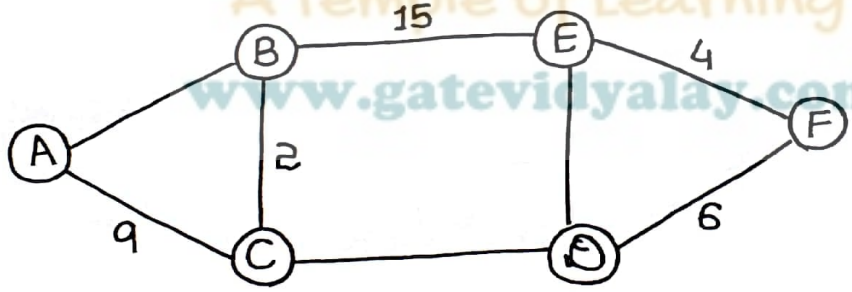
Correct option is (B).

Question:

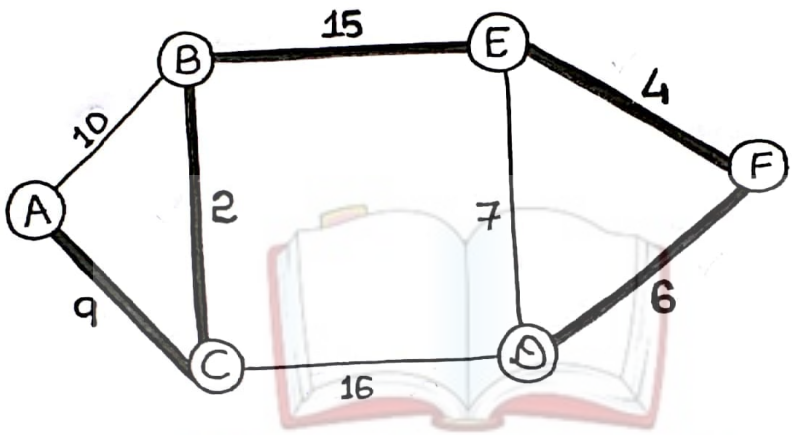
The graph shown below has 8 edges with distinct integer edge weights. The minimum spanning tree (MST) is of weight 36 and contains the edges $\{(A,C), (B,C), (B,E), (E,F), (D,F)\}$. The edge weights of only those edges which are in the MST are given in the figure shown below.

The minimum possible sum of weights of all 8 edges of this graph is _____.

(GATE-2015)



Solution:



Minimum possible sum of weights of all 8 edges of this graph

$= 10 + 16 + 7 + 36$

$= 69$ units