Binary Trees

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**Definition:**

- Binary tree is a special type of tree data structure in which every node can have at most 2 children, i.e., each node can have either 0 child or 1 child or 2 children, but not more than 2 children.

**Example:**

![Binary Tree Diagram]

- Node with 2 children
- Node with 0 child
- Node with only 1 child
Unlabelled Binary Trees:

- A binary tree is unlabelled if its nodes are not assigned any label.

**Example:**

![Diagram of an unlabelled binary tree with three nodes connected in a tree structure.]

Number of different binary trees possible with 'n' unlabelled nodes

\[ \frac{2^n C_n}{n+1} \]
Illustration:

With 3 unlabelled nodes, number of different binary trees possible:

\[ \frac{2 \times 3 \binom{3}{3}}{3 + 1} = \frac{6 \binom{3}{3}}{4} = 5 \]

Thus, with 3 unlabelled nodes, 5 different binary trees are possible.
Labelled Binary Trees:

- A binary tree is labelled if every node is assigned a label.

Example:

![Diagram of a labelled binary tree with nodes A, B, and C, where A is the root, B and C are children of A.]

Number of binary trees possible with \( n \) labelled nodes is given by:

\[
\text{Number of binary trees} = \frac{2^n \binom{n}{n} \times n!}{n+1}
\]
Illustration:

With 3 labelled nodes, number of different binary trees possible

\[ \frac{2 \times 3 \times C_3 \times 3!}{3 + 1} = \frac{6 \times 3!}{4} = 30 \]

Example: The structure will give \(3! = 6\) different labelled structure.
Types of Binary Trees

1. Rooted Binary Tree:

- A rooted binary tree is one that has a root node and every node has at most 2 children.

Example:

```
       A
      / \  
     B   C
    /     \
   D     E
```


2. **Full/Strictly Binary Tree**:

- A full/strictly binary tree is one in which every node has either 0 or 2 children.

**Example:**

```
  A
 / \  /
B   C  D  E
|   /  /   /
|  G  F  H  I
```

Not Full/Strictly Binary Tree

```
  A
 / \  /
B   C  D  E
|   /  /   /
|  G  F  H  I
```

Full/Strictly Binary Tree
3. Complete / Perfect Binary Tree:

- A complete / perfect binary tree is one in which every internal node has exactly two children, and all the leaf nodes are at the same level.

- Example:

```
Not a complete / perfect Binary Tree

Complete / Perfect Binary Tree
```
4. Almost Complete Binary Tree:

- An almost complete binary tree is one in which all the levels are completely filled except possibly the last level and the last level must be strictly filled from left to right.

- Example:

```
Not almost complete Binary Tree

Almost complete Binary Tree
```
5. Skewed Binary Trees:

- A skewed binary tree is one where all the nodes except one node have one and only one child. The remaining node has no children.

OR

A skewed binary tree is a binary tree of n nodes such that its depth is n-1.

- Example: